Name: _____

School: _____

University of Houston
High School Contest – Spring 2008
Algebra I Test

Directions: You have 50 minutes to complete this exam. Calculators are not permitted. Choose the correct answer for each question by clearly and boldly circling the answer choice. There is no penalty for guessing. In the case of a tie, students' work will be used to determine the winner – so show all work clearly on either your exam or on scrap paper. Write your name and school on top of each page in the blanks provided.

1. Give one factor of the polynomial. The variables used as exponents represent positive integers.
$-24u^{3m}v^{n}+4u^{2m}v^{2n}+48v^{3n}u^{m}$

a. $(2u^{m}-3v^{n})$
b. This polynomial cannot be factored.
c. $u^{3m}v^{3n}$
d. $4u^{3m}v^{3n}$
e. $(-2u^{m}-3v^{n})$

2. Perform the indicated operation. Simplify completely. The variable in the exponents represents positive integers.
$$\frac{x^{2k}-9}{3x^{k+3}+9x^{3}} \div \frac{x^{k+2}-3x^{2}}{6x^{k+5}}$$

a. $2x^{k}$
b. $\dfrac{2x^{k}(x^{k}+3)}{x^{k}+9}$
c. $\dfrac{(x^{k}-3)^{2}}{18x^{k+6}}$
d. $\dfrac{1}{2}x^{k-6}$
e. $2k$

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3. Replace k in the following trinomial by a number that makes the trinomial a perfect square trinomial.
$ks^2 - 24s + 9$

a. 4
b. 3
c. 16
d. 9
e. 12

4. Simplify. $1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{x}}}}$.

a. $\dfrac{3x+2}{x}$
b. $\dfrac{5x+3}{3x+2}$
c. x
d. $\dfrac{4x+1}{x}$
e. $\dfrac{2x+6}{x+4}$

5. Give a real solution to the equation $12x^3 + 16x^2 - 5x - 3 = 0$ given that $-\dfrac{3}{2}$ is a root.

a. $-\dfrac{1}{3}$
b. 0
c. No other real solutions exist.
d. 2
e. $-\dfrac{1}{2}$

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6. One day you went canoeing. You learned that the current in the river was 2 miles per hour. If it took you the same amount of time to travel 10 miles downstream as 2 miles upstream, determine the speed at which your canoe would travel in still water.

a. 3 miles per hour
b. 0.5 mile per hour
c. 5 miles per hour
d. 1 mile per hour
e. 2 miles per hour

7. Given $2x^2 + 3x + c = 0$, find all values of c that will result in the equation having no real number solution.

a. It is not possible to find all values of c.
b. $c < \frac{9}{8}$
c. $c \geq 1$
d. $c > \frac{9}{8}$
e. $c < 1$

8. Simplify.

$$\frac{\sqrt{90x^4y}}{\sqrt{2x^5y^5}}$$

a. $\frac{3\sqrt{5}}{y^2}$
b. $\frac{3\sqrt{5x}}{2xy^2}$
c. $\frac{15}{xy^2}$
d. $\frac{3\sqrt{x^5y^6}}{x^3y^5}$
e. $\frac{3\sqrt{5x}}{xy^2}$

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9. A storeowner wants to set up a rectangular display area outside his store. He will use one side of the building (which is 25 feet long) as part of one side of the display area. He has 230 linear feet of fencing material to use to fence in the display area. Write a function which expresses the total area of the display area in terms of the length x of the side of the display area opposite the building.

a. $A(x) = \frac{255}{2}x + x^2$

b. $A(x) = \frac{255}{2}x - x^2$

c. $A(x) = \frac{253}{2}x^2$

d. $A(x) = \frac{253}{2}x$

e. $A(x) = 230x - 25$

10. Solve for x.

$$\left|x^2 + 6x + 1\right| = 8$$

a. $x = -7, x = 1$
b. $x = -7, x = -3$
c. $x = -7, x = -3, x = 1$
d. $x = -3, x = 1$
e. $x = 1$

11. Give a factor of $x^4 - 10{,}000$ over real numbers.

a. This polynomial cannot be factored over real numbers.
b. $x^2 + 10$
c. $x^4 - 100$
d. $x - 10$
e. $x^4 + 100$

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12. Simplify.

$$\frac{\sqrt{x}}{\sqrt{2}-\sqrt{x}}$$

a. $\frac{\sqrt{2x}+x}{2-x}$
b. $\sqrt{x}$
c. $\frac{2\sqrt{x}+x}{4-x}$
d. $\frac{\sqrt{x}-x}{2+x}$
e. $\frac{\sqrt{2x}-x}{2-x}$

13. Larry, a chemistry student, needs 10% acetic acid solution for a chemistry experiment. He realizes he only has 5% and 20% acetic acid solutions available. He decides to make the 10% solution by combining the 5% and 20% solutions. How many liters of the 5% solution must he add to 8 liters of the 20% solution to get a solution that is 10% acetic acid?

a. 16.5 liters
b. 32 liters
c. 16 liters
d. 8.5 liters
e. 24 liters

14. For exercise, Carrie and Amanda run from their home to a nearby park. Carrie runs 3 ½ miles per hour. Amanda runs 7 ¼ miles per hour. When Amanda arrives at the park, Carrie is $5\ 5/8$ miles away. How far is the park from their house?

a. 11.78125 miles
b. 10.875 miles
c. 9.125 miles
d. 1.5 miles
e. 1.625 miles

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15. A ball is dropped from a height of 128 feet. Its height s, in feet, at time t, in seconds, is given by $s(t) = -16t^2 + 96t + 128$. When will the ball hit the ground?

a. after 2 seconds
b. after $6 + 2\sqrt{17}$ seconds
c. after $2\sqrt{2}$ seconds
d. after $3 + \sqrt{17}$ seconds
e. after 4 seconds

Problems 16 - 25 start on the next page!!

(16) Solve the following system for x and y:

$$\frac{7}{x} + \frac{2}{y} = 5$$
$$\frac{1}{x} + \frac{4}{y} = -3$$

(a) Infinitely many solutions
(b) $\{(\frac{15}{7}, \frac{15}{13})\}$
(c) $\{(\frac{15}{13}, \frac{15}{7})\}$
(d) No solutions
(e) $\{(1, -1)\}$
(f) None of the above.

(17) Jim can't remember the last digit of his best friend's phone number. He plans to just dial the number and guess the last digit. What's the probability that he'll dial the correct number in the first 2 tries?

(a) $\frac{1}{10}$
(b) $\frac{1}{45}$
(c) $\frac{17}{10}$
(d) $\frac{1}{5}$
(e) $\frac{2}{5}$
(f) None of the above.

(18) Jeanini bought a ring in 1990 for \$200. By 1995 it had lost 3% of its value. In 2000 it was worth 33% more than in 1995. By 2005 it had lost 10% of its value from 5 years earlier. What was the ring worth in 2005 (in nearest whole dollars)?

(a) \$232
(b) \$229
(c) \$227
(d) \$213
(e) \$221
(f) None of the above.

(19) Find the real number solutions: $\sqrt{3x+1}+\sqrt{x-4}=3$.

(a) 5

(b) 8

(c) 5,8

(d) No real number solutions.

(e) 4,5,8

(f) None of the above.

(20) Find the remainder of x^8+1 divided by $x-1$.

(a) 1

(b) -1

(c) 2

(d) -2

(e) 0

(f) None of the above.

(21) Find the largest number that always divides the difference of the squares of two consecutive even numbers.

(a) 1

(b) 2

(c) 3

(d) 4

(e) None of the above.

(22) Jim buys pixie sticks at 4 for 10 cents. He sells them at 4 for 15 cents. How many does he have to sell in order to make a profit of \$1.00?

(a) 80

(b) 40

(c) 8

(d) 125

(e) 25

(f) None of the above.

(23) Instead of walking along the adjacent sides of a rectangular field, Joe took a shortcut along the diagonal and saved a distance equal to half the length of the longer side. Find the ratio of the longer side to the shorter side of the field.

(a) 5:3
(b) 3:2
(c) 2:3
(d) 5:2
(e) 4:3
(f) None of the above.

(24) Write 25.25 in base 2.

(a) 1011.01
(b) 1101.01
(c) 11001.001
(d) 1101.001
(e) 11001.01
(f) None of the above.

(25) There are 500 red and blue balls in a jar. 1% of the balls are blue. Some red balls, but no blue balls, are withdrawn. Then 2% of the balls are blue. How many balls are still in the jar?

(a) 490
(b) 495
(c) 485
(d) 250
(e) 245
(f) None of the above.

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University of Houston
High School Mathematics Contest
Geometry Exam – Spring 2008

Directions:
You have 50 minutes to complete this exam. Calculators are not permitted. Choose the correct answer for each question, and write the letter (A, B, C, D or E) corresponding to that answer in the blank to the right of each question. There is no penalty for guessing. In the case of a tie, students' work will be used to determine the winner – so show all work clearly on either your exam or on scrap paper. Write your name and school on top of each page in the blanks provided.
Note: Geometric figures in the problems may not be drawn to scale.

1. Given $\overline{AB}$ with endpoint $A(-2, 6)$ and midpoint $M(-5, 1)$, find the coordinates of endpoint B.

(A) $\left(-\frac{3}{2}, -\frac{5}{2}\right)$

(B) $\frac{5}{3}$

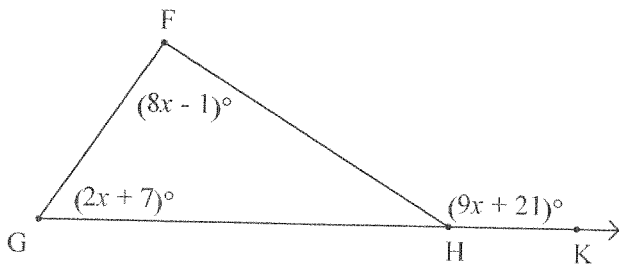
(C) $(-8, -4)$

(D) $\left(-\frac{7}{2}, \frac{7}{2}\right)$

(E) $(-12, 8)$

1. _____

2. ΔFGH is shown below, with point K located on $\overrightarrow{GH}$. If $m\angle G = (2x+7)^\circ$, $m\angle F = (8x-1)^\circ$, and $m\angle FHK = (9x+21)^\circ$, find the measure of $\angle FHG$.



2. _____

(A) 15° (B) 24° (C) 37° (D) $93\frac{9}{19}^\circ$ (E) 156°

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3. Find the measure of an exterior angle of a regular convex decagon.

(A) 36°

(B) 144°

(C) 18°

(D) 45°

(E) 135°

3. _____

4. Find the area of an equilateral triangle with altitude 12cm.

(A) $27\sqrt{3}$ cm^2

(B) $48\sqrt{3}$ cm^2

(C) $96\sqrt{3}$ cm^2

(D) 72 cm^2

(E) $72\sqrt{3}$ cm^2

4. _____

5. Given the following conditional statement:

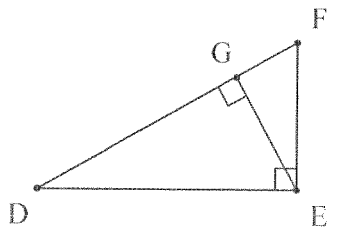
If a quadrilateral has four right angles, then it is a square.

Determine the truth value of the inverse, contrapositive, and converse, respectively.

(A) False, True, False
(B) False, False, False
(C) True, True, True
(D) True, False, True
(E) True, True, False

5. _____

6. Right triangle DEF is shown below with altitude $\overline{EG}$. If the length of $\overline{EG}$ is $2\sqrt{10}$ cm and the length of $\overline{GF}$ is 5 cm, find the length of $\overline{DF}$.



(A) $\left(7\sqrt{10}\right)$ cm

(B) 20 cm

(C) $\sqrt{65}$ cm

(D) $\left(\frac{5\sqrt{10}}{4}+5\right)$ cm

(E) 13 cm

6. _____

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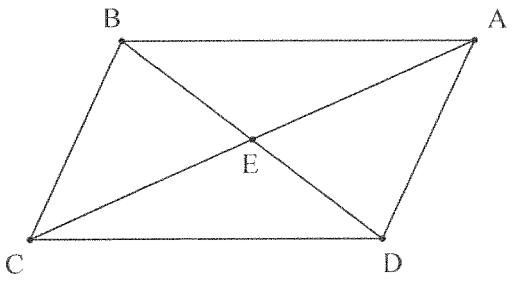
7. Which of Objects 2-5 below could represent the image of Object 1 as the result of a single rotation (with no other transformations applied)?

Object 1 Object 2 Object 3 Object 4 Object 5

7. _____

(A) Objects 2 and 3 only (B) Objects 4 and 5 only (C) Object 4 only
(D) Objects 2, 3, and 4 only (E) Objects 2, 3, 4, and 5

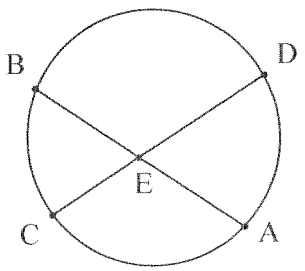
8. In the quadrilateral below, $\overline{BA} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{AD}$, $m\angle CBD = (6x-5)^\circ$, $m\angle ABD = (3x+4)^\circ$, and $m\angle BAD = (7x+5)^\circ$. Find the measure of $\angle ADB$.



8. _____

(A) 13° (B) 98° (C) 11° (D) 37° (E) 61°

9. In the circle below, chord $\overline{CD}$ bisects chord $\overline{AB}$ at point E. If the measure of $\overline{CE}$ is nine less than twice the measure of $\overline{BE}$, and the measure of $\overline{ED}$ is six more than the measure of $\overline{EA}$, find the measure of chord $\overline{CD}$. *Note: All units are in centimeters.*



9. _____

(A) 24 cm (B) 15 cm (C) 12 cm (D) 6 cm (E) 3 cm

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10. Which of the following polyhedra have 9 faces?

10. _____

I. Nonagonal prism
II. Heptagonal prism
III. Triangular prism
IV. Octagonal pyramid
V. Nonagonal pyramid

(A) IV only (B) II and IV only (C) III only
(D) I and V only (E) II, III, and IV only

11. The sum of the interior angles of a regular convex polygon is 1620°. Find the measure of one of its interior angles.

11. _____

(A) 162°
(B) $147\frac{3}{11}^\circ$
(C) 135°
(D) 11°
(E) 9°

12. Given ΔABC and ΔDEF with $\overline{AC} \cong \overline{FD}$, $\angle A \cong \angle D$, and $\overline{BC} \cong \overline{FE}$, what theorem or postulate would follow directly to prove that $\Delta ABC \cong \Delta DEF$?

12. _____

(A) Side-Side-Side
(B) Angle-Angle-Side
(C) Side-Angle-Side
(D) Angle-Side-Angle
(E) The triangles are not necessarily congruent

13. If 5 and 9 are each side lengths of an acute triangle, find a number that could be the third side length.

13. _____

(A) 5 (B) 8 (C) 3 (D) $2\sqrt{14}$ (E) 14

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14. In right triangle DEF, $\overline{DE}$ is the hypotenuse and G is the midpoint of $\overline{DE}$. If $GF = 6x - 2$ and $GE = 4x + 8$, find the length of $\overline{DE}$.

14. _____

(A) 5

(B) 14

(C) $\frac{104}{5}$

(D) 56

(E) Cannot be determined

15. Draw ΔACD with B located between A and C. Given that $\angle C \cong \angle ADB$, $AB = 4$ and $AD = 6$, find BC.

15. _____

(A) 5

(B) $2\sqrt{6}$

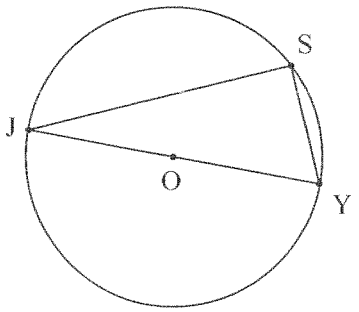
(C) 8

(D) 9

(E) 10

16. ΔSJY is inscribed in circle O having diameter $\overline{JY}$. If $OY = 5$ and $SY = 3$, find the cosine of $\angle SJY$.

16. _____



(A) $\frac{4}{5}$

(B) $\frac{3}{10}$

(C) $\frac{3\sqrt{91}}{91}$

(D) $\frac{10\sqrt{109}}{109}$

(E) $\frac{\sqrt{91}}{10}$

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17. If the surface area of a sphere is $\frac{9\pi}{4}$ cm^2, find its volume.

17. _____

(A) $\frac{9\pi}{16}$ cm^3

(B) 36π cm^3

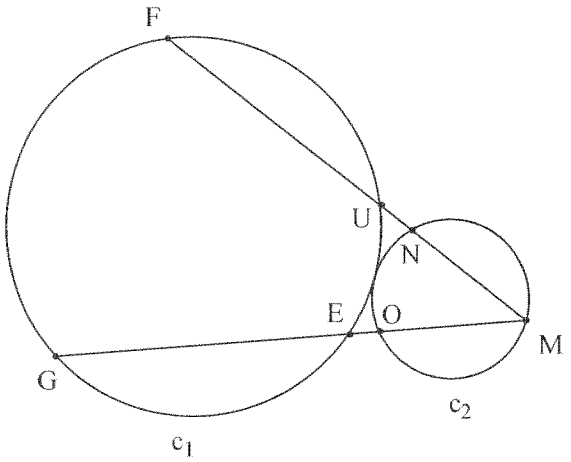
(C) $\frac{9\pi}{2}$ cm^3

(D) $\frac{32\pi}{81}$ cm^3

(E) $\frac{27\pi}{16}$ cm^3

18. Circles c_1 and c_2 are externally tangent as shown below. If $m\widehat{GF} = 125^\circ$ and $m\widehat{NO} = 84^\circ$, find $m\widehat{UE}$.

18. _____



(A) 62.5°
(B) 41°
(C) 125°
(D) 42°
(E) 84°

19. If the ratio of the lateral area of two similar hexagonal prisms is 4:121, find the ratio of their volumes.

19. _____

(A) 4 : 121
(B) 8 : 1331
(C) 2 : 11
(D) 16 : 14641
(E) 64 : 1771561

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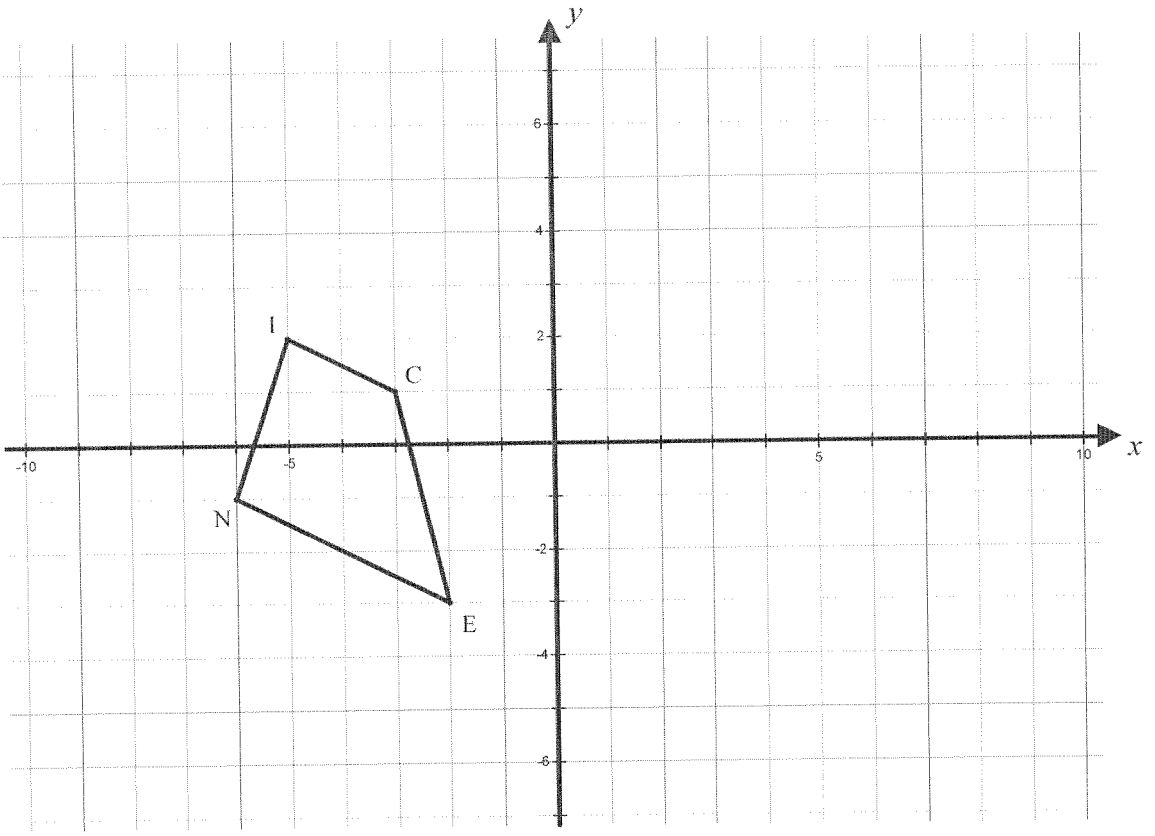
20. If quadrilateral *NICE* is reflected across the line $y = x + 2$, give the coordinates of each of the vertices of the image $N'I'C'E'$.



(A) $N'(-3,-4)$, $I'(0,-3)$, $C'(-1,-1)$, $E'(-5,0)$

(B) $N'(-6,5)$, $I'(-5,2)$, $C'(-3,3)$, $E'(-2,7)$

(C) $N'(10,-1)$, $I'(9,2)$, $C'(7,1)$, $E'(6,-3)$

(D) $N'(6,-1)$, $I'(5,2)$, $C'(3,1)$, $E'(2,-3)$

(E) $N'(-1,\ -6)$, $I'(2,-5)$, $C'(1,-3)$, $E'(-3,-2)$

20. _____

Exam is continued on the next page...

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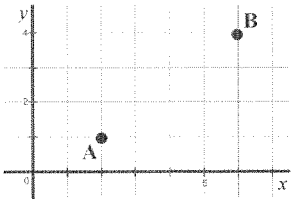
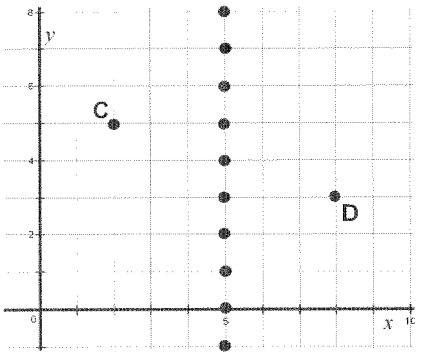
21. In Taxicab Geometry, the shortest distance between two points is defined as the minimum number of city blocks a taxi would need to travel to get from one point to the other, assuming square blocks of equal size and all streets oriented only horizontally and vertically.

In the diagram at the right, for example, the distance between the points $A(2,1)$ and $B(6,4)$ is 7.

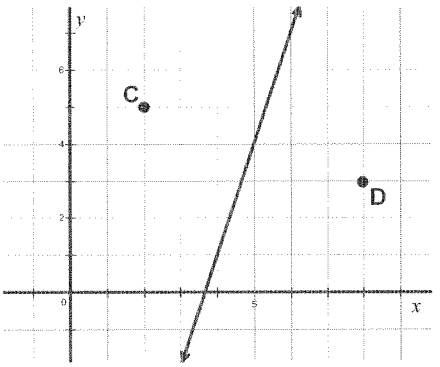


Using this Taxicab definition of distance, draw the set of points that are equidistant from points $C(2,5)$ and $D(8,3)$.

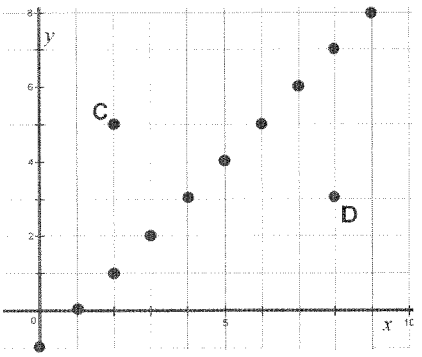
(A)



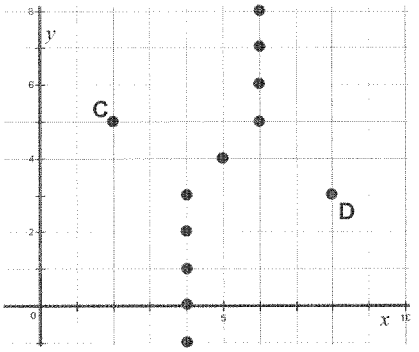
(B)



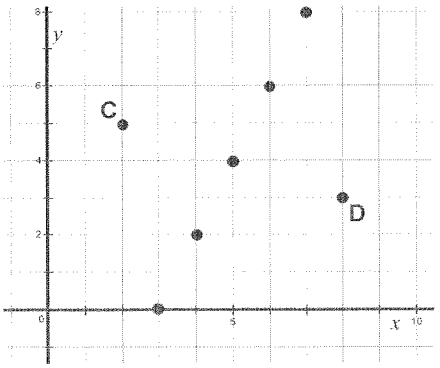
(C)



(D)



(E)



21. _____

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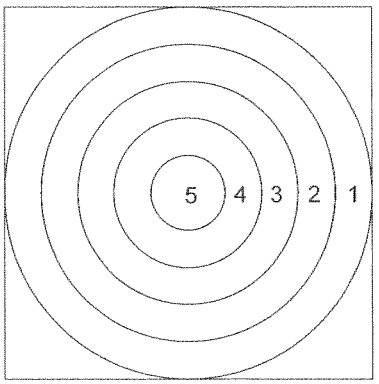
22.



The square dartboard above is circumscribed about the largest of five concentric circles, and the ratio of the radii of the circles is 1:2:3:4:5. The numbers on the dartboard above represent the point values obtained for hitting that particular ring on the board. If a dart is thrown at the dartboard, what is the probability of hitting a ring worth an odd number of points? (Assume that all darts thrown are randomly distributed within the outer square.)

(A) $\frac{3}{5}$

(B) $\frac{3\pi}{20}$

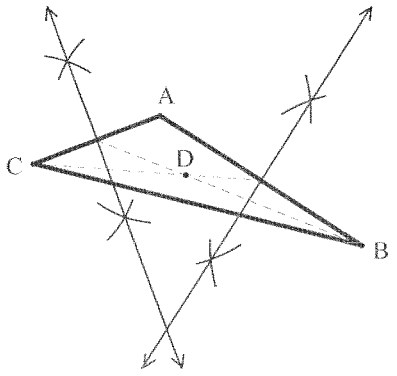
(C) $\frac{\pi}{10}$

(D) $\frac{\pi}{4}$

(E) $\frac{3\pi}{5}$

22. _____

23. A construction has been performed on triangle ABC below, with all relevant lines and arcs of circles shown. Point D represents the _____ of the triangle.



(A) Incenter
(B) Circumcenter
(C) Orthocenter
(D) Centroid
(E) Center of dilation

23. _____

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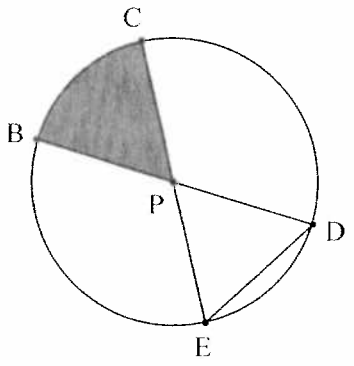
24. Diameters $\overline{BD}$ and $\overline{CE}$ of circle P are shown below, along with chord $\overline{ED}$. If $\angle PED$ measures 70° and the radius of the circle measures 12 cm, find the area of the shaded sector.

24. _____



(A) $\frac{8\pi}{3}$ cm^2

(B) 16π cm^2

(C) 22π cm^2

(D) 32π cm^2

(E) 56π cm^2

25. Find the number of diagonals in a convex 16-gon. (Hint: Look for a pattern.)

25. _____

(A) 104

(B) 112

(C) 120

(D) 208

(E) 2520

26. Triangle ABC is drawn on a coordinate plane with vertices $A(-5,-3)$, $B(-3,7)$, and $C(4,3)$. Find the length of the altitude drawn from vertex B.

26. _____

(A) $\sqrt{65}$

(B) $\frac{3}{2}$

(C) $\frac{\sqrt{221}}{2}$

(D) $\sqrt{117}$

(E) $2\sqrt{13}$

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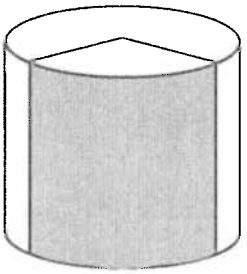
27. A cylindrical can of radius 9 cm and height 15 cm is shown below, and the gray shaded area represents a sticker that is placed on the lateral surface of the can. A central angle measuring 100° is shown on the upper base of the cylinder, and the arc subtended by the central angle coincides with the edge of the sticker. Find the area of the sticker.



(A) $\frac{45\pi}{2}$ cm^2

(B) $\frac{675\pi}{2}$ cm^2

(C) 135 cm^2

(D) 270π cm^2

(E) 75π cm^2

27. _____

28. A spherical scoop of ice cream with radius 4 cm fits entirely inside a right circular ice cream cone so that the scoop of ice cream is tangent to the plane containing the circular base of the cone, and the point of tangency P is the center of the base. (In other words, the ice cream reaches the opening of the cone – but does not show above the cone itself.) The height of the cone is 16 cm. Find the lateral area of the cone.

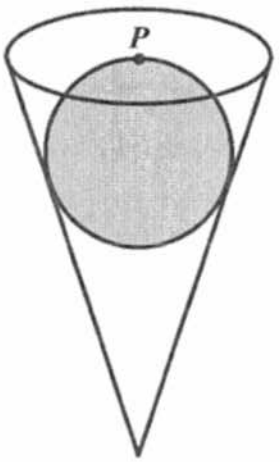
(A) $16\sqrt{17}\,\pi$ cm^2

(B) $128\sqrt{2}\,\pi$ cm^2

(C) 96π cm^2

(D) 192π cm^2

(E) $\frac{512\pi}{3}$ cm^2

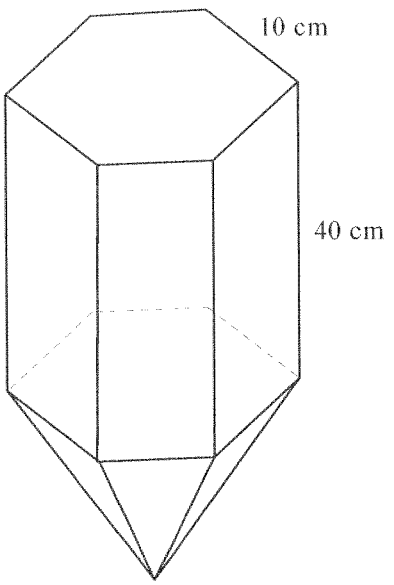


28. _____

Name: _____ School: _____

29. A pencil-like shape is formed by joining a regular right hexagonal prism and a regular right hexagonal pyramid, as shown in Figure 1 below. The regular hexagon (representing the base of both the prism and the pyramid) has a side length of 10 cm. The prism has height 40 cm, and the pyramid has slant height $\sqrt{219}$ cm.

The solid is shown again in Figure 2, with a bold segment on the base of the prism which joins the midpoints of opposite sides of the hexagon. Find the area of the cross-section formed if the object is sliced through this bold segment, perpendicular to the base and passing through the apex of the pyramid.



Slant height of pyramid $= \sqrt{219}$ cm

Figure 1

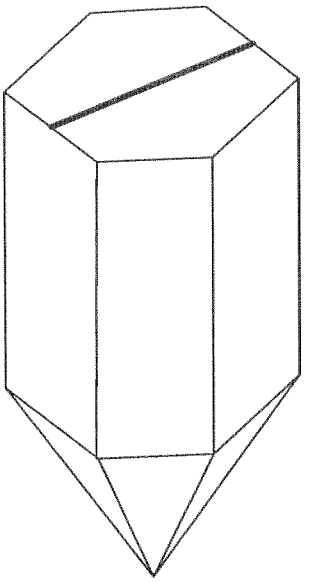


Figure 2

(A) $920\ \text{cm}^2$

(B) $\left(400\sqrt{3}+15\sqrt{73}\right)\text{cm}^2$

(C) $520\sqrt{3}\ \text{cm}^2$

(D) $\left(800+10\sqrt{119}\right)\text{cm}^2$

(E) $460\sqrt{3}\ \text{cm}^2$

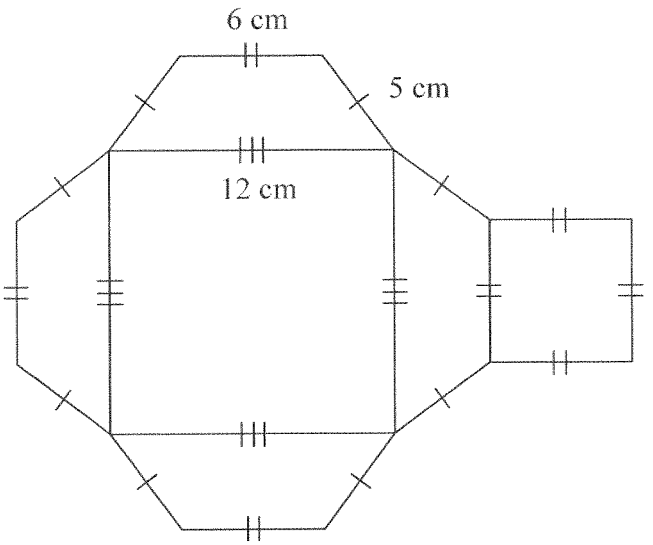
29. _____

Exam is continued on the next page...

Name: _____ **School:** _____

30. The net below is comprised of a square of side length 12 cm, surrounded by four congruent isosceles trapezoids with bases measuring 12 and 6 cm, and legs measuring 5 cm. One of the trapezoids shares a side with a square of side length 6 cm. Find the volume of the solid formed by this net.

30. _____



(A) 324 cm^3

(B) 192 cm^3

(C) 336 cm^3

(D) $84\sqrt{7}$ cm^3

(E) 420 cm^3

END OF EXAM ☺

(Make sure all answer blanks are filled in with the letter of the correct answer.)

ALGEBRA II EXAM

UNIVERSITY OF HOUSTON
2008 HIGH SCHOOL MATHEMATICS CONTEST
FEBRUARY 16, 2008

Name:

School:

Score:

Circle your answers.

(1) Find i^{673}
   (a) i
   (b) $-i$
   (c) 1
   (d) -1
   (e) None of the above.

(2) $log_8 x = -\frac{1}{3}$. Solve for x.
   (a) 512
   (b) No solutions.
   (c) 2
   (d) $\frac{8}{3}$
   (e) $\frac{1}{2}$
   (f) None of the above.

(3) In 3 years John will be half his mother's current age. John's grandfather is twice as old as John's mom. The grandfather is 48. How old is John?
   (a) 18
   (b) 9
   (c) 6
   (d) 12
   (e) 8
   (f) None of the above.

(4) A boat has a speed of 20 miles per hour (mph) in still water. In a stream that has a current of 10 mph it travels a certain distance downstream and returns. Find the ratio of the time the round trip takes to the time it would take in still water.
   (a) 3:2
   (b) 1:2
   (c) 3:4
   (d) 4:3
   (e) 5:4
   (f) None of the above.

(5) Simplify $log_2 24 - log_2 \frac{3}{2}$
   (a) 36
   (b) $2log_2 36$
   (c) 4
   (d) 2
   (e) $log_2 22.5$
   (f) None of the above.

(6) Find the coordinates of the center of the circle

$$x^2 + y^2 - 8x + 4y + 16 = 0.$$

(a) $(4, -2)$
(b) $(4, 2)$
(c) $(-4, -2)$
(d) $(-4, 2)$
(e) $(2, -4)$
(f) None of the above.

(7) At the conclusion of a party, 55 handshakes are exchanged among the guests. Assuming that each guest shook hands with each of the others, find the number of guests n at the party.

(a) 11
(b) 10
(c) 9
(d) 8
(e) None of the above.

(8) Each edge of a cube is increased by 40%. What percentage does the surface area of the cube increase?

(a) 40%
(b) 196%
(c) 156%
(d) 96%
(e) 256%
(f) None of the above.

(9) Simplify: $(2x^2 - 7x - 4)^{-1}(2x^2 + 7x + 3)(x^2 + 3x - 28)(x^2 - 9)^{-1}$

(a) $\frac{x+3}{x+4}$
(b) $\frac{x+7}{x-3}$
(c) $\frac{2x+1}{x-4}$
(d) 1
(e) $\frac{x+4}{x+3}$
(f) None of the above.

(10) How many solutions in non-negative integers does the inequality $x + y < 50$ have? The solutions (x, y) and (y, x) are considered to be different when $x \neq y$.

(a) 1200
(b) 1225
(c) 1250
(d) 1275
(e) 2300
(f) None of the above.

(11) Simplify: $\sqrt[3]{81} - \sqrt[3]{\frac{1}{9}}$.

(a) $\frac{8\sqrt[3]{3}}{3}$
(b) $\frac{\sqrt[3]{3}}{24}$
(c) $\frac{-8\sqrt[3]{3}}{49}$
(d) $\frac{\sqrt[3]{24}}{3}$
(e) $\sqrt[3]{9}$
(f) None of the above.

(12) Simplify: $\frac{xy-y^2}{xy-x^2} - \frac{x^2-y^2}{xy}$.

(a) $\frac{x}{y}$
(b) $\frac{-x}{y}$
(c) $\frac{-y}{x}$
(d) $\frac{y}{x}$
(e) $\frac{xy-x^2}{xy}$
(f) None of the above.

(13) Which triples can NOT be the lengths of the sides of a right triangle.

I. $\{12, 5, 13\}$.

II. $\{21, 29, 20\}$.

III. $\{19, 27, 20\}$.

IV. $\{11, 12, 5\}$.

(a) II,III,IV

(b) II,IV

(c) I,II,III,IV

(d) II,III

(e) III,IV

(f) None of the above.

(14) If x workers can do a job in d days, then how many days will it take $x + y$ workers?

(a) $\frac{xy}{d+y}$

(b) $\frac{yd}{x+d}$

(c) $\frac{xd}{x+y}$

(d) $\frac{xd}{d+y}$

(e) $\frac{d}{x+y}$

(f) None of the above.

(15) If $\frac{1}{a+x} = \frac{1}{a} + \frac{1}{x}$, and a is a non-zero constant, how many real number solutions will this equation have?

(a) 0

(b) 1

(c) 2

(d) 4

(e) Infinitely many.

(f) None of the above.

(16) Solve the following system for x and y:

$$\frac{7}{x} + \frac{2}{y} = 5$$
$$\frac{1}{x} + \frac{4}{y} = -3$$

(a) Infinitely many solutions
(b) $\{(\frac{15}{7}, \frac{15}{13})\}$
(c) $\{(\frac{15}{13}, \frac{15}{7})\}$
(d) No solutions
(e) $\{(1, -1)\}$
(f) None of the above.

(17) Jim can't remember the last digit of his best friend's phone number. He plans to just dial the number and guess the last digit. What's the probability that he'll dial the correct number in the first 2 tries?

(a) $\frac{1}{10}$
(b) $\frac{1}{45}$
(c) $\frac{17}{10}$
(d) $\frac{1}{5}$
(e) $\frac{2}{5}$
(f) None of the above.

(18) Jeanini bought a ring in 1990 for \$200. By 1995 it had lost 3% of its value. In 2000 it was worth 33% more than in 1995. By 2005 it had lost 10% of its value from 5 years earlier. What was the ring worth in 2005 (in nearest whole dollars)?

(a) \$232
(b) \$229
(c) \$227
(d) \$213
(e) \$221
(f) None of the above.

(19) Find the real number solutions: $\sqrt{3x+1}+\sqrt{x-4}=3$.

(a) 5

(b) 8

(c) 5,8

(d) No real number solutions.

(e) 4,5,8

(f) None of the above.

(20) Find the remainder of x^8+1 divided by $x-1$.

(a) 1

(b) -1

(c) 2

(d) -2

(e) 0

(f) None of the above.

(21) Find the largest number that always divides the difference of the squares of two consecutive even numbers.

(a) 1

(b) 2

(c) 3

(d) 4

(e) None of the above.

(22) Jim buys pixie sticks at 4 for 10 cents. He sells them at 4 for 15 cents. How many does he have to sell in order to make a profit of \$1.00?

(a) 80

(b) 40

(c) 8

(d) 125

(e) 25

(f) None of the above.

(23) Instead of walking along the adjacent sides of a rectangular field, Joe took a shortcut along the diagonal and saved a distance equal to half the length of the longer side. Find the ratio of the longer side to the shorter side of the field.
   (a) 5:3
   (b) 3:2
   (c) 2:3
   (d) 5:2
   (e) 4:3
   (f) None of the above.

(24) Write 25.25 in base 2.
   (a) 1011.01
   (b) 1101.01
   (c) 11001.001
   (d) 1101.001
   (e) 11001.01
   (f) None of the above.

(25) There are 500 red and blue balls in a jar. 1% of the balls are blue. Some red balls, but no blue balls, are withdrawn. Then 2% of the balls are blue. How many balls are still in the jar?
   (a) 490
   (b) 495
   (c) 485
   (d) 250
   (e) 245
   (f) None of the above.